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Stochastic resonance in a double quantum dot system

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Stochastic resonance (SR) is theoretically investigated for a double quantum dot system represented by two discrete levels in respective wells. The system is driven by a periodic signal and a white noise source with variable amplitude, and thus displays an improved output signal-to-noise ratio, a characteristic signature of SR.

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In recent years, phenomena such as time-dependent tunneling and stochastic resonance (SR) have received considerable attention [1–3]. Calculation of tunneling time needed for an electron to traverse a potential barrier was reported as early as in the 1980s [4], and later on other works on time dependence of tunneling using an effective Schrödinger equation [5] and nonequilibrium Green's function [6] were reported. Since the pioneering work of Kramers [7] for potential barrier crossing using thermal activation, the phenomenon of SR has become very relevant in a wide range of scientific fields such as physics, chemistry, biology, climatology, environmental sciences, etc. [1,2].

The recent developments in nanofabrication techniques have provided artificial analog of atoms, molecules, and crystals. Quantum dots (QDs) behave as artificial atoms [8], and two strongly coupled QDs display properties of artificial molecules [9]. Photon-assisted tunneling current through a single quantum dot with an effectively continuous level spectrum was measured [10]. In another interesting work for the double-dot system, the dc current measurement resolved the resonances between energy levels of both the dots [11]. The time-dependent resonant tunneling via two discrete states in the double-dot system was investigated theoretically [12,13]. An important property of the mesoscopic quantum system subjected to time-varying field is the spatial and temporal coherence of their electronic states, which can give rise to numerous interesting phenomena [14,15]. Very recently coherent signal amplification in bistable nanomechanical oscillators and optical bistable system by stochastic resonance were demonstrated [3]

In this work we consider a system of double QDs characterized by only two nondegenerate and weakly coupled electronic levels with energies ϵ_1 and ϵ_2. We further assume that in the absence of any external perturbation the level 1 is occupied and level 2 is empty. This manipulation can be achieved by adjusting a bias voltage on the sample. Also, coupling between two quantum dots is very weak so no tunneling of the electron takes place. Typically such a system is represented by two discrete states in two different wells (Fig. 1), which is an ideal two-state system and is quite suitable for studying the phenomenon of SR. The generic model of SR describes the motion of a particle in a double-well potential—a situation very much similar to the system under consideration. The essence of SR phenomenon is counter-intuitive, i.e., adding a certain amount of noise to the input of system can actually increase the output signal-to-noise ratio (SNR) for a signal passing through the nonlinear medium, and the optimal improvement occurs at a certain noise strength. For the double-QD system, we concentrate on the two states only and disregard other states that are allowed in the neighborhood of resonance. The energies of resonant states differ by $\epsilon_0=\epsilon_2-\epsilon_1$. Also, under these conditions the transport through the system via bias voltage and Coulomb blockade tunneling is negligible. When a time-dependent field $E(t)=E_0\cos(\omega t)$ is applied via gate electrodes to the dots (E_0 is the amplitude, ω is frequency), the on-site energies oscillate against each other [given by $\epsilon(t)$, as defined after Eq. (3)], which is the requirement for the SR.

The dynamical evolution of the two-state system with resonant states $|1\rangle$ and $|2\rangle$ is governed by the following master equation for the whole system statistical operator:

$$\frac{\partial\rho}{\partial t}=-\frac{i}{\hbar}[H(t),\rho]+Ł_r\rho+Ł_w\rho, \tag{1}$$

with the time-dependent tunneling Hamiltonian

$$H(t)=H_0(t)+H_{int}(t), \tag{2}$$

in which H_0 is given by

$$H_0(t)=\frac{1}{2}\epsilon(t)(|2\rangle\langle2|-|1\rangle\langle1|), \tag{3}$$

and $\epsilon(t)=\epsilon_0+\wp E(t)$, $E(t)=E_0\cos(\omega t)$ is the externally applied field and $\wp$ is constant such that $\wp E(t)$ has dimension of energy. H_{int} introduces coupling between dots and causes mixing of the states $|1\rangle$ and $|2\rangle$ of the system and is given by

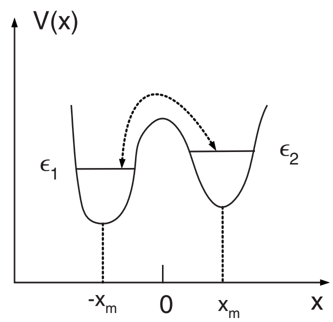

FIG. 1. Schematics of two quantum dots showing energy levels in the respective potential well.

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$$H_{int}(t) = \zeta(|2\rangle\langle 1| + |1\rangle\langle 2|), \tag{4}$$

where ζ is the coupling strength between the two dots. In Eq. (1) $Ł_r\rho$ is Liouville superoperator for reservoir tunneling and $Ł_w\rho$ describes interdot electron relaxation. The term $Ł_r\rho$ can be negligible at the very weak-tunneling output leads, i.e., quantum dots are weakly coupled to the electron reservoirs. Coulomb blockade suppresses the movement of electrons through the double dot at low bias voltages. If one works nearly at such conditions, then SR phenomenon could be realized. Inclusion of $Ł_r\rho$ will essentially contribute to the loss mechanism and should not disturb the SR phenomenon when its magnitude is small.

It should be noted that the Liouvillian $Ł_w\rho$ is not always needed to describe the effect of classical noise, but it can be used in certain problems (a generalized approach, e.g., in random phase-noise process) due to the fact that noise is causing fluctuations in the system, and from the fluctuation-dissipation theorem it will give rise to dissipation in the system. The Liouvillian $L_w\rho$ contains both incoherent (rate-dominated relaxation—caused by, say, classical noise) dissipation as well as dissipation for coherent (oscillatory dominated) tunneling. By the choice of parameters coherent tunneling can be suppressed considerably, so the dominant relaxation mechanism of $L_w\rho$ will be incoherent relaxation rates. These rates are modulated due to the presence of a periodic signal, as we will see in the following [Eq. (13)]. Physically, the Liouvillian $L_w\rho$ is describing the interaction of a system with heat bath oscillators. The system loses its excitation to heat bath oscillators (via microscopic interactions: $\Sigma_k[\hat{b}_k(|2\rangle\langle 1|)+\text{H.c.}]$; where $\hat{b}_k, \hat{b}_k^\dagger$ are the bath operators, and hence $L_w\rho$ is explicitly given by Eq. (8.2.8) of Ref. [16]), but acquires some equivalent random noise in the process of dissipation (another manifestation of fluctuation-dissipation theorem) [16]. The knowledge of the damping coefficient leads to determine spectral function of statistical fluctuations and the noise correlator. Thus in Eq. (1), $L_w\rho$ provides a general method of microscopically modeling a heat bath of infinite collection of harmonic oscillators, which are interacting with the system and giving rise to dissipation terms as explicitly mentioned in the following Eq. (5). For very weak dissipative coherent tunneling, the $L_w\rho$ approach is equivalent to modeling the interaction of the system with classical noise through Langevin equation [16].

In order to describe the particle (electron) dynamics for this two-state system one can use the rate-equation approach (rate-dominated relaxation—caused by classical noise) as is usually adopted to study the generic SR model. Since there is a weak interaction between the dots, we use a density-matrix (ρ) approach using master equation (1),

$$\dot{\rho}_{11} = -\Gamma_{11}\rho_{11} + \Gamma_{22}\rho_{22} - i(\zeta\rho_{12} - \zeta^*\rho_{21}),$$

$$\dot{\rho}_{22} = -\Gamma_{22}\rho_{22} + \Gamma_{11}\rho_{11} + i(\zeta\rho_{12} - \zeta^*\rho_{21}),$$

$$\dot{\rho}_{12} = -\Gamma_{12}\rho_{12} - i\epsilon(t)\rho_{12} - i\zeta(\rho_{11} - \rho_{22}),$$

$$\dot{\rho}_{21} = -\Gamma_{12}\rho_{21} + i\epsilon(t)\rho_{21} + i\zeta^*(\rho_{11} - \rho_{22}), \tag{5}$$

where ρ_{11} and ρ_{22} denote the probabilities for the particle (electron) to be in the left and right dots, respectively, and $\rho_{12}=\rho_{21}^*$ are the off-diagonal density matrix elements. The terms proportional to $\Gamma_{11}(t)$ and $\Gamma_{22}(t)$ describe the transitions to and from between the states $|1\rangle$ and $|2\rangle$, respectively. We define $\Gamma_{12}(t)=[\Gamma_{11}(t)+\Gamma_{22}(t)]/2$. The potential constituted by the double dots has a double-well structure with minima located at $\pm x_m$. The height of the potential barrier is measured from the lower state $|1\rangle$ and is given by ΔV. In the absence of coupling between dots, the first two equations become decoupled from the remaining two and exactly match with the two-state rate equations describing the generic SR phenomenon [1–3]. If one applies a Gaussian white noise with zero mean and autocorrelation function,

$$\langle \xi(t)\xi(0)\rangle = 2D\delta(t), \tag{6}$$

then in the absence of periodic signal, the noise-induced hopping between local equilibrium states will take place with Kramers rate [1]

$$r_k \propto \frac{1}{\sqrt{2}\pi} \exp\left(-\frac{\Delta V}{D}\right), \tag{7}$$

where D is the noise strength. In the presence of a periodic signal the relative separation of the states in the two wells changes and the presence of appropriate (optimum) noise strength causes random-switching frequency r_k to agree closely with the signal frequency ω and the particle makes the transition to the other state with higher probability.

In the absence of noise and the periodic signal, if the interaction between dots (ζ) is small compared to ϵ_0, the electron is highly localized in one or the other potential well, inhibiting any transition. On the other hand, if the system is driven at a frequency (or subharmonic) corresponding to the condition $\omega=\sqrt{\epsilon_0^2+4\zeta^2}$, photon-assisted tunneling takes place. We avoid this condition for the study of SR in the following and perform a transformation [17] on the density-matrix equations such that

$$\bar{\rho}_{21} = \rho_{21} \exp\left(-i\int_{-\infty}^{t} dt' \wp E(t')\right). \tag{8}$$

By doing so the explicit time dependence is eliminated (except in the transition rates) and the time dependence is introduced in the interaction parameter of dots as

$$\bar{\zeta}(t) = \zeta \exp\left(i\int_{-\infty}^{t} dt' \wp E(t')\right), \tag{9}$$

and the equation for the off-diagonal elements can be written as

$$\dot{\bar{\rho}}_{21} = [i\epsilon_0 - \Gamma_{12}(t)]\bar{\rho}_{21} + i\bar{\zeta}^*(t)(\rho_{11} - \rho_{22}), \tag{10}$$

with $\bar{\rho}_{21}=(\bar{\rho}_{12})^*$. The expansion of $\zeta(t)$ in the Fourier series is given by

$$\bar{\zeta}(t) = \zeta \sum_{n=-\infty}^{\infty} J_n(\wp E_0/\omega)\exp(in\omega t). \tag{11}$$

In the weak-coupling limit of QDs and weak signal field, Eq. (9) can be manipulated to obtain $\bar{\rho}_{21}$ in the lowest nonvanishing order in ζ, and substituting that solution into the first two equations of Eq. (5) we obtain

$$\dot{\rho}_{11} = -\left(\Gamma_{11}(t) + \Gamma_{22}(t) + \frac{4i\epsilon_0\zeta J_1^2}{\omega^2 - \epsilon_0^2 + \Gamma_{12}^2/4}\right)\rho_{11} + \Gamma_{22}(t) + \frac{2i\epsilon_0\zeta J_1^2}{\omega^2 - \epsilon_0^2 + \Gamma_{12}^2/4} + i\zeta J_0(\rho_{12}^0 - \rho_{21}^0),$$

$$\dot{\rho}_{22} = -\left(\Gamma_{11}(t) + \Gamma_{22}(t) + \frac{4i\epsilon_0\zeta J_1^2}{\omega^2 - \epsilon_0^2 + \Gamma_{12}^2/4}\right)\rho_{22} + \Gamma_{11}(t) + \frac{2i\epsilon_0\zeta J_1^2}{\omega^2 - \epsilon_0^2 + \Gamma_{12}^2/4} - i\zeta J_0(\rho_{12}^0 - \rho_{21}^0). \tag{12}$$

Equations (12) are essentially the rate equations derived from Eq. (5) under the assumption that ζ is small and the interdot transition rates (Γ_{ij}; $i,j=1,2$) are governed by the noise only. For such noise-induced transition rates in the presence of a periodically modulated signal we can assume an Arrhenius-type function [1–3]

$$\Gamma_{11,22}(t) = r_k \exp\left(\pm\frac{E_0 x_m}{D}\cos(\omega t)\right). \tag{13}$$

For the weak modulation $E_0 x_m \ll D$ and under adiabatic approximation,

$$\Gamma_{11}(t) + \Gamma_{22}(t) \cong 2r_k\left[1 + \frac{1}{2}\left(\frac{E_0 x_m}{D}\right)^2\cos^2(\omega t) + \cdots\right]. \tag{14}$$

Equation (11) may now be integrated (assuming $\zeta\epsilon_0 \ll \omega, \Gamma_{12}$) and, to first order in field amplitude, we obtain

$$\rho_{22}(t;x_0,t_0) = \left[e^{-2r_k(t-t_0)}\left(\rho_{22}(t_0) - \frac{r_k - i\zeta J_0(\rho_{21}^0 - \rho_{12}^0)}{2r_k} - \frac{r_k E_0 x_m}{D\sqrt{\omega^2 + 4r_k^2}}\cos(\omega t_0 - \phi)\right) + \frac{r_k - i\zeta J_0(\rho_{21}^0 - \rho_{12}^0)}{2r_k} + \frac{r_k E_0 x_m}{D\sqrt{\omega^2 + 4r_k^2}}\cos(\omega t - \phi)\right], \tag{15}$$

where $\phi \equiv \tan^{-1}(\omega/2r_k)$. The quantity $\rho_{22}(t;x_0,t_0)$ represents the conditional probability that $x(t)$ is in state 2 at time t, given that the state at time t_0 was x_0 (which may be x_m or $-x_m$). Retaining higher powers of E_0 in Eq. (13) leads to higher harmonics in the power spectrum. One can obtain the autocorrelation function as [17]

$$\begin{aligned}\langle x(t)x(t+\tau)|x_0,t_0\rangle = {} & x_m^2\rho_{22}(t+\tau|x_m,t)\rho_{22}(t|x_0,t_0) \\ & - x_m^2\rho_{22}(t+\tau|-x_m,t)\rho_{11}(t|x_0,t_0) \\ & - x_m^2\rho_{11}(t+\tau|x_m,t)\rho_{22}(t|x_0,t_0) \\ & + x_m^2\rho_{11}(t+\tau|-x_m,t)\rho_{11}(t|x_0,t_0).\end{aligned} \tag{16}$$

The term $x_m^2\rho_{22}(t+\tau|-x_m,t)\rho_{11}(t|x_0,t_0)$ represents the situation that at t_0 the particle is at x_0 (note that the system takes on the discrete values $\pm x_m$ at all times), at t it is at $-x_m$, and at $t+\tau$ it is at x_m. In the limit of $t_0 \to -\infty$, the autocorrelation function greatly simplifies to

$$\begin{aligned}\langle x(t)x(t+\tau)|x_0,t_0\rangle_{t_0\to-\infty} &= \langle x(t)x(t+\tau)\rangle \\ &= x_m^2 e^{-2r_k\tau}\left(1 - \frac{\zeta^2 J_0^2(2\rho_{21}^{0,im})^2}{r_k^2} - \frac{4r_k^2E_0^2x_m^2}{D^2(\omega^2+4r_k^2)}\times\cos^2(\omega t-\phi)\right) + \frac{4r_k^2E_0^2x_m^2}{D^2(\omega^2+4r_k^2)} \\ &\quad \times\{\cos(\omega\tau) + \cos[\omega(2t+\tau) - 2\phi]\}.\end{aligned} \tag{17}$$

The power spectrum is the Fourier transform of this autocorrelation function. This function is ensemble averaged first and then taken Fourier transform to give [17]

$$\begin{aligned}\langle S(\Omega)\rangle_t &= \int_{-\infty}^{\infty}\langle\langle x(t)x(t+\tau)\rangle\rangle_t e^{-i\Omega\tau}d\tau \\ &= x_m^2\left(1 - \frac{4\zeta^2J_0^2(\rho_{21}^{0,im})^2}{r_k^2} - \frac{2r_k^2E_0^2x_m^2}{D^2(\omega^2+4r_k^2)}\right)\left(\frac{4r_k}{4r_k^2+\Omega^2}\right) \\ &\quad + \frac{2r_k^2E_0^2x_m^2}{D^2(\omega^2+4r_k^2)}[\delta(\Omega+\omega) + \delta(\Omega-\omega)],\end{aligned} \tag{18}$$

where the ensemble average is defined as

$$\langle\langle x(t)x(t+\tau)\rangle\rangle_t = \frac{\omega}{2\pi}\int_0^{2\pi/\omega}\langle x(t)x(t+\tau)\rangle dt. \tag{19}$$

One can also use $S(\Omega)$ as a one-sided t-averaged power spectrum in which $S(\Omega)$ is defined for positive Ω only, i.e.,

$$\begin{aligned}\langle S(\Omega)\rangle &= \langle S(\Omega)\rangle_t + \langle S(-\Omega)\rangle_t \\ &= \left(1 - \frac{4\zeta^2J_0^2(\rho_{21}^{0,im})^2}{r_k^2} - \frac{2r_k^2E_0^2x_m^2}{D^2(\omega^2+4r_k^2)}\right)\left(\frac{8r_k}{4r_k^2+\Omega^2}\right) \\ &\quad + \frac{4\pi r_k^2E_0^2x_m^2}{D^2(\omega^2+4r_k^2)}[\delta(\Omega-\omega)].\end{aligned} \tag{20}$$

The spectrum contains two parts. The signal output (the last term) is a δ function at signal frequency. The broadband noise output (first term) is a Lorentzian profile centered at $\Omega=0$, which has three contributions: one with no signal, the other due to an interaction between quantum dots, and the last one representing correction due to signal on the noise. The correction factor has the effect of an overall reduction of the broadband noise power which is eventually transferred into the δ-function spike. Thus the effect of the signal is to transfer power from broadband noise to the δ-function spike and thus increase the signal-to-noise ratio. There is another mechanism (proportional to J_0^2) reducing the noise power because of very small but finite coupling between the two quantum dots.

The signal-to-noise ratio in this situation is given by

$$R = \frac{\pi r_k E_0^2 x_m^2}{2D^2}\left(1 - \frac{4\zeta^2 J_0^2(\rho_{21}^{0,im})^2}{r_k^2} - \frac{2r_k^2 E_0^2 x_m^2}{D^2(\omega^2 + 4r_k^2)}\right)^{-1}. \tag{21}$$

The plot of R with respect to D is given in Fig. 2. Under the assumption that the interaction (ζ) between dots is small, the electron initially will be localized in one of the QDs and hence $\rho_{21}^{0,im} \sim 0$, so the second term in large parentheses of Eq. (20) will not contribute. From Fig. 2 we observe that for very small D (compared to ΔV) the exponential terms falls to zero rapidly so R is nearly zero. On the contrary, for very large D, the exponential term reaches to 1 but the term D^2 in the denominator makes R again zero. In the midway values of D, there is the maximum situated near $D_M \sim \Delta V$. The effect of an increase in the amplitude of the signal on R is shown in curve B of Fig. 2.

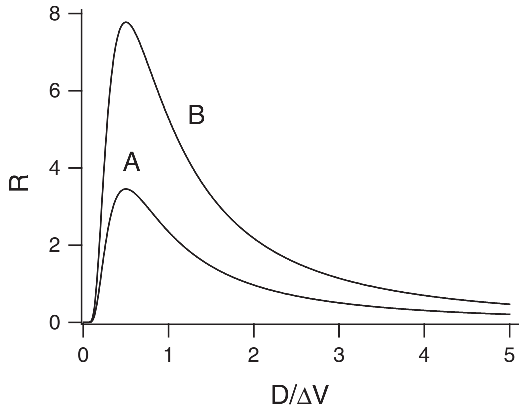

FIG. 2. Signal-to-noise ratio (R) as a function of normalized noise amplitude ($D/\Delta V$). Curves A and B are for two different signal amplitudes ($E_0 x_m/\Delta V$) such that the amplitude in B is 1.5 times that of A.

To summarize, in this work we studied the phenomenon of stochastic resonance in a system of weakly coupled quantum dots driven by a weak signal and the Gaussian white noise source and found the existence of a cooperative phenomenon, i.e., incoherent noise power is feeding into coherent signal. We have used a natural simplification of the double-well situation in terms of the discrete two-state system. By providing modulating signal we vary the energy levels in the two wells periodically rather than tilting the potential wells, which is usually the case in the phenomenon of SR. Under weak coupling between the two quantum dots, as well as weak coupling to electron reservoirs, it is possible to observe the phenomenon of SR in this system, which is useful in quantum computing [18].

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